Tomasz Placek

# Branching space-times

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Typically, the past (and the present) is settled, but the future is not (it is open)

Historical modalities:

It is (already) settled that ...

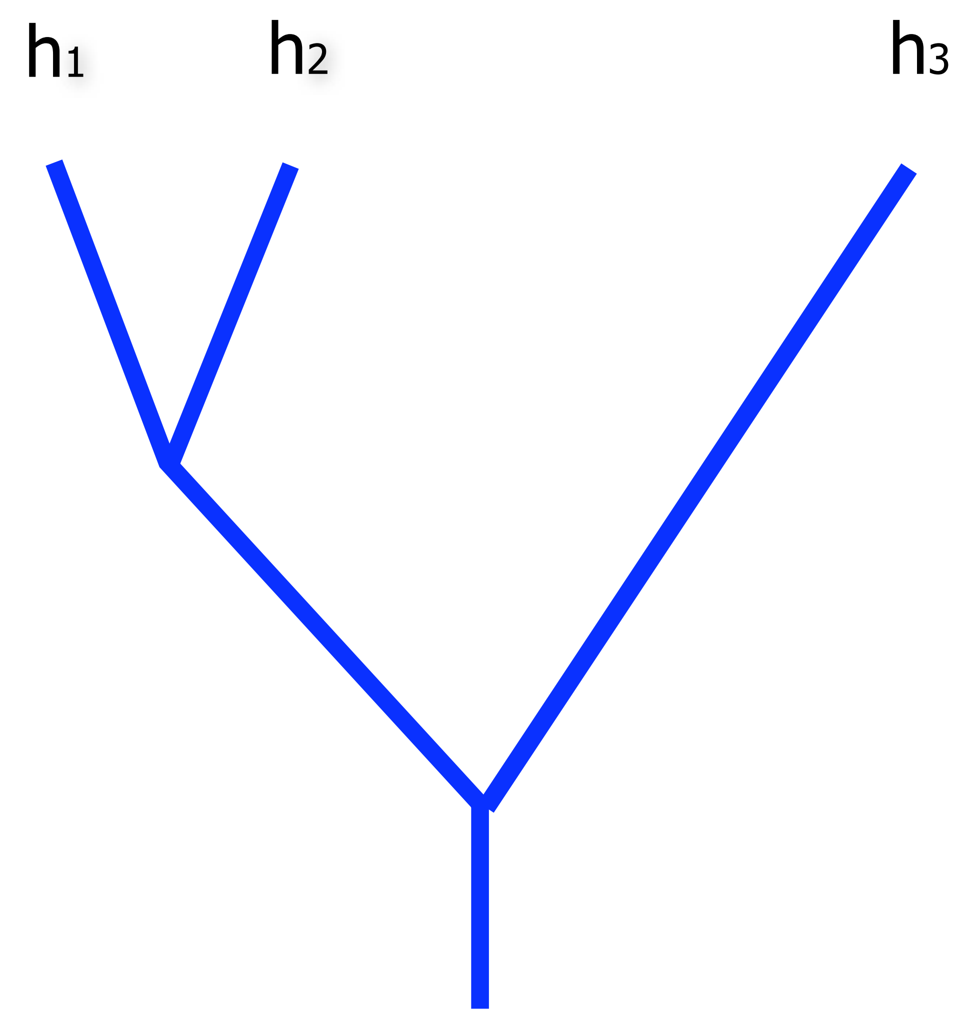
It is (still) possible that ...

Aristotle's tomorrow's sea battle

Diodoros Chronos's Master Argument

# Branching time (Prior, Kripke, Thomason)

# Model BT: time + possibilities



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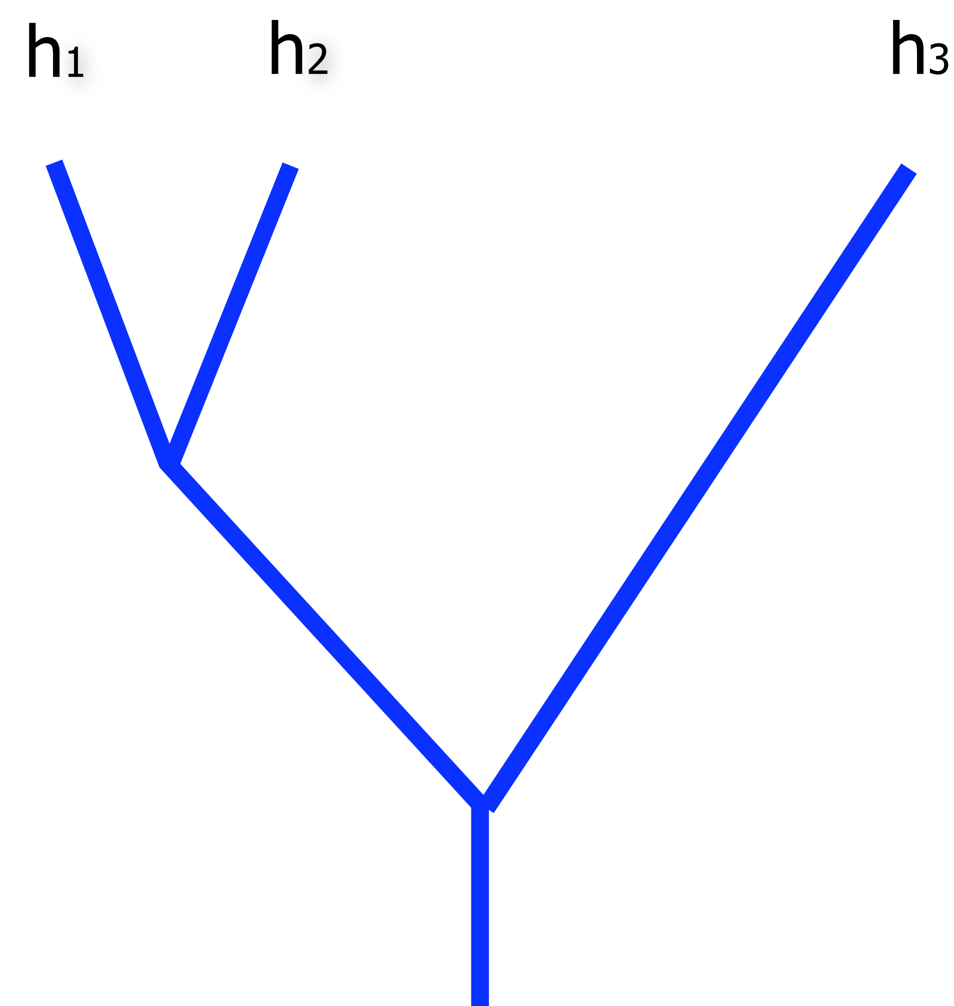
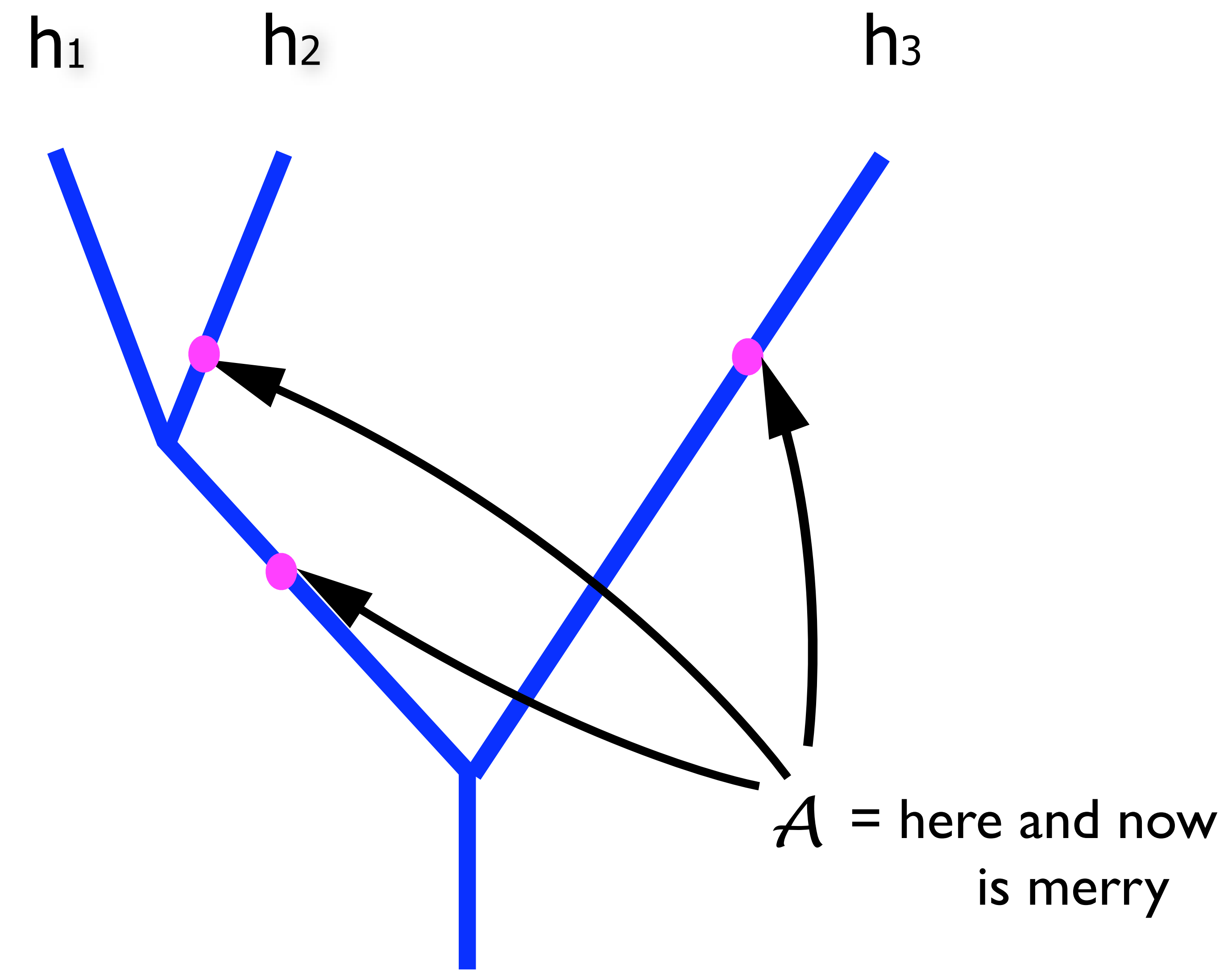



Non-empty partially ordered set, with no backward branching.
Histories identified with maximal chains in the base set.

# Semantical model



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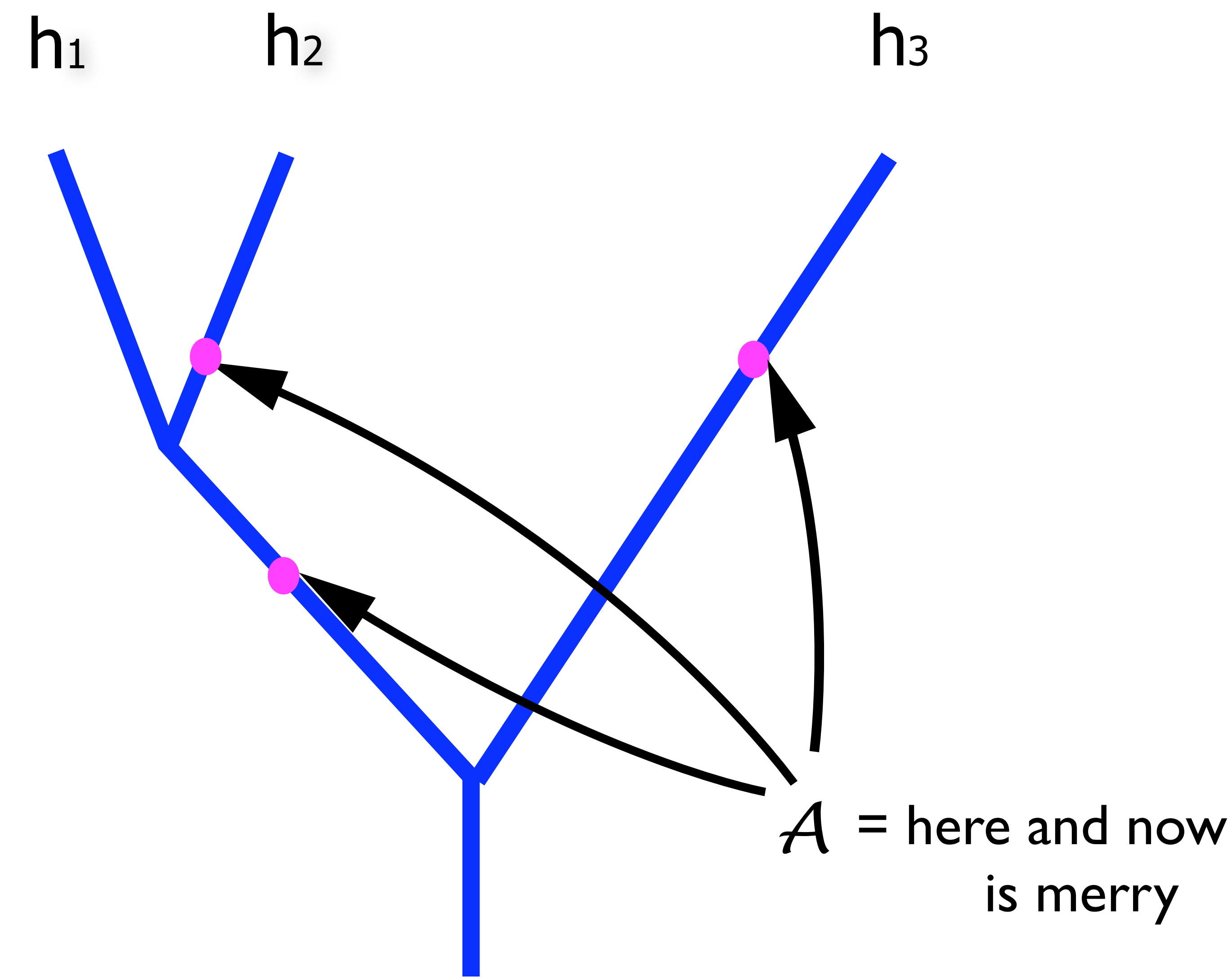
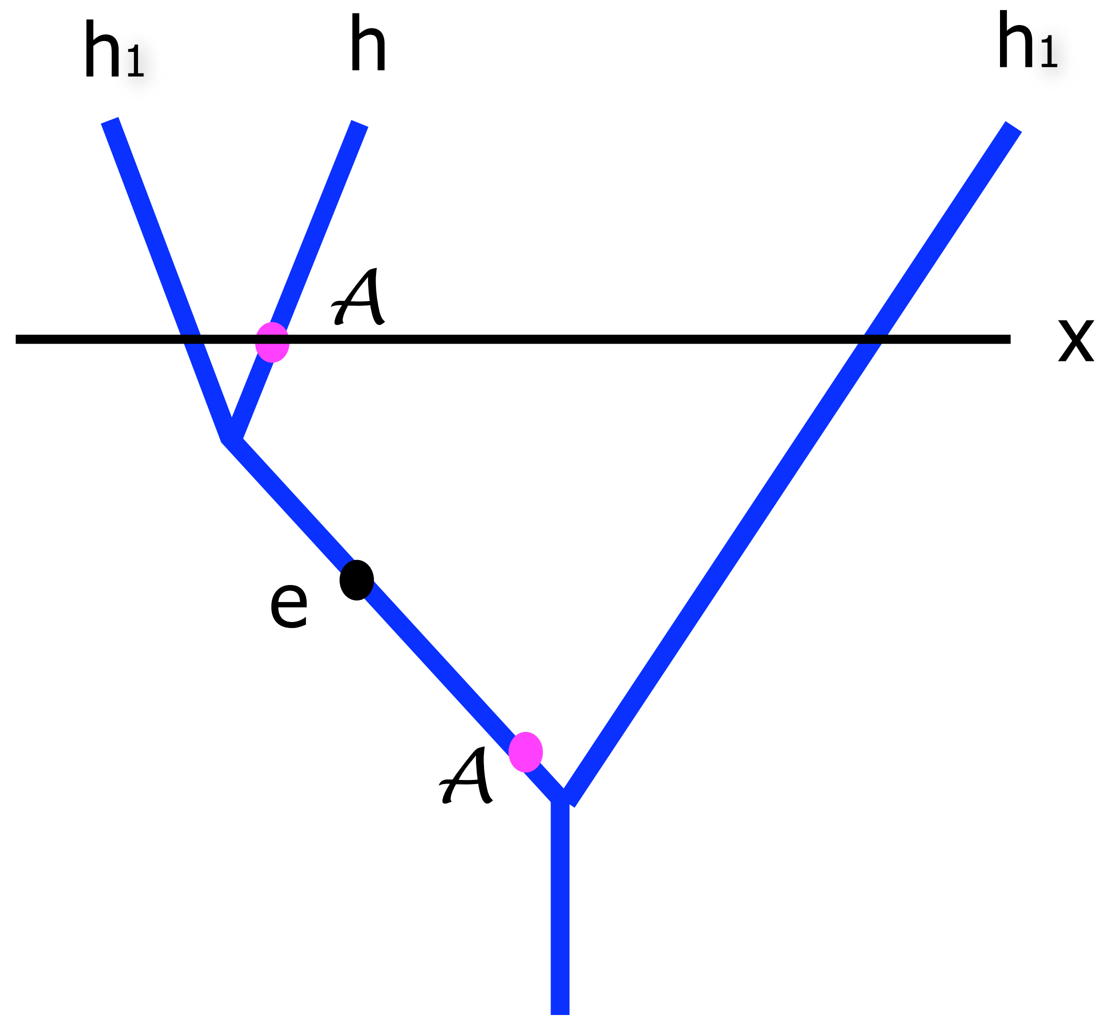



BT model + interpretation function *I* assigning atomic formulas to events

Novelty of Prior/Thomason : sentences are true/false at <event, history> pairs.

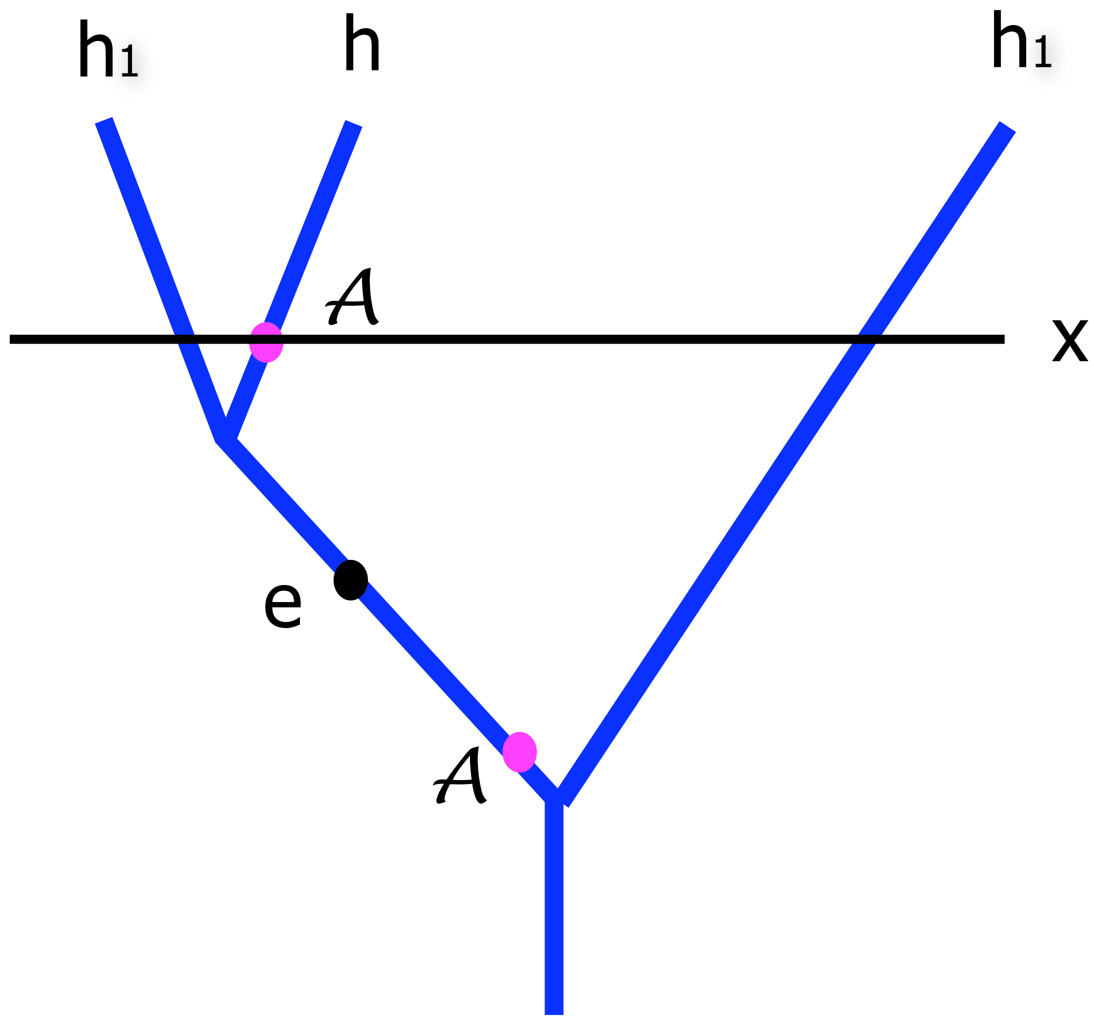



$e/h \models A$ iff $e \in \mathcal{I}(A)$ for $A$ an atomic formula;

$e/h \models Will: A$ iff $\exists e' > e: e'/h \models A$;

$e/h \models Was: A$ iff $\exists e' < e: e'/h \models A$,

where $e/h$ is a pair $\langle e, h \rangle$ such that $e \in h$




$$e/h \models Poss\colon A \text{ iff } \exists h'\colon e \in h' \wedge e/h' \models A;$$

$$e/h \models Sett\colon A \text{ iff } \forall h'\colon e \in h' \rightarrow e/h' \models A;$$

# Branching space-times - Belnap 1992

possible histories have spatial and relativistic aspects

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What should replace the BT notion of history as maximal chain?

Minkowski space-time: the relation "x lies in the future light cone of y" is a partial order

Assume that a base set W for BST is partially ordered.

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Add modality. How to interpret forks?

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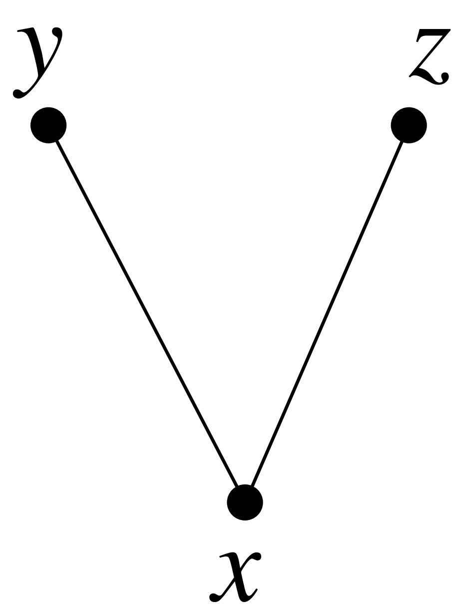
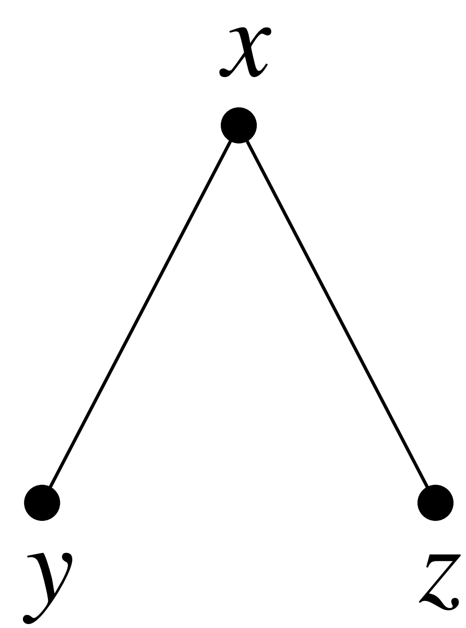
BST




| | |
|---|---|
| spatiotemporal int. | spatiotemporal int. |
| modal int. | ––––––––––– |

# History

A subset A of W is upward directed if every two elements of A have an upper bound in A

A is a maximal upward directed subset of W if every proper superset of A is not upward directed.

History of W is a maximal upward directed subset of W

# History is intended to be like Minkowski space-time

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That any two histories divide at some e follows from Prior Choice Principle:

Let $O$ be a chain in $W$ such that $O \subset h_1$, but $O \cap h_2 = \emptyset$ for some histories $h_1, h_2$.
Then there is an $e$ such that $e <_\forall O$ and $h_1 \perp_e h_2$.

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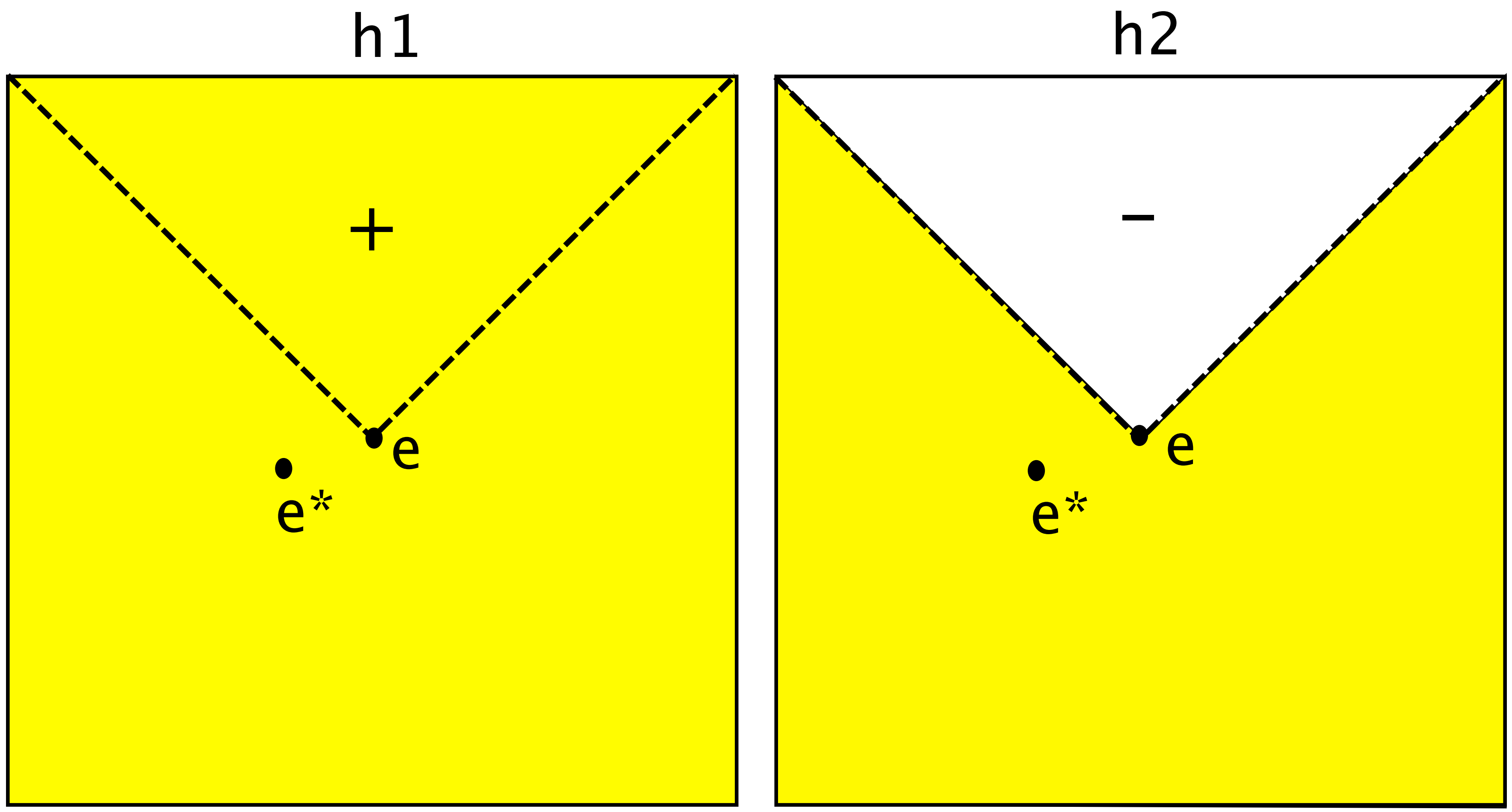
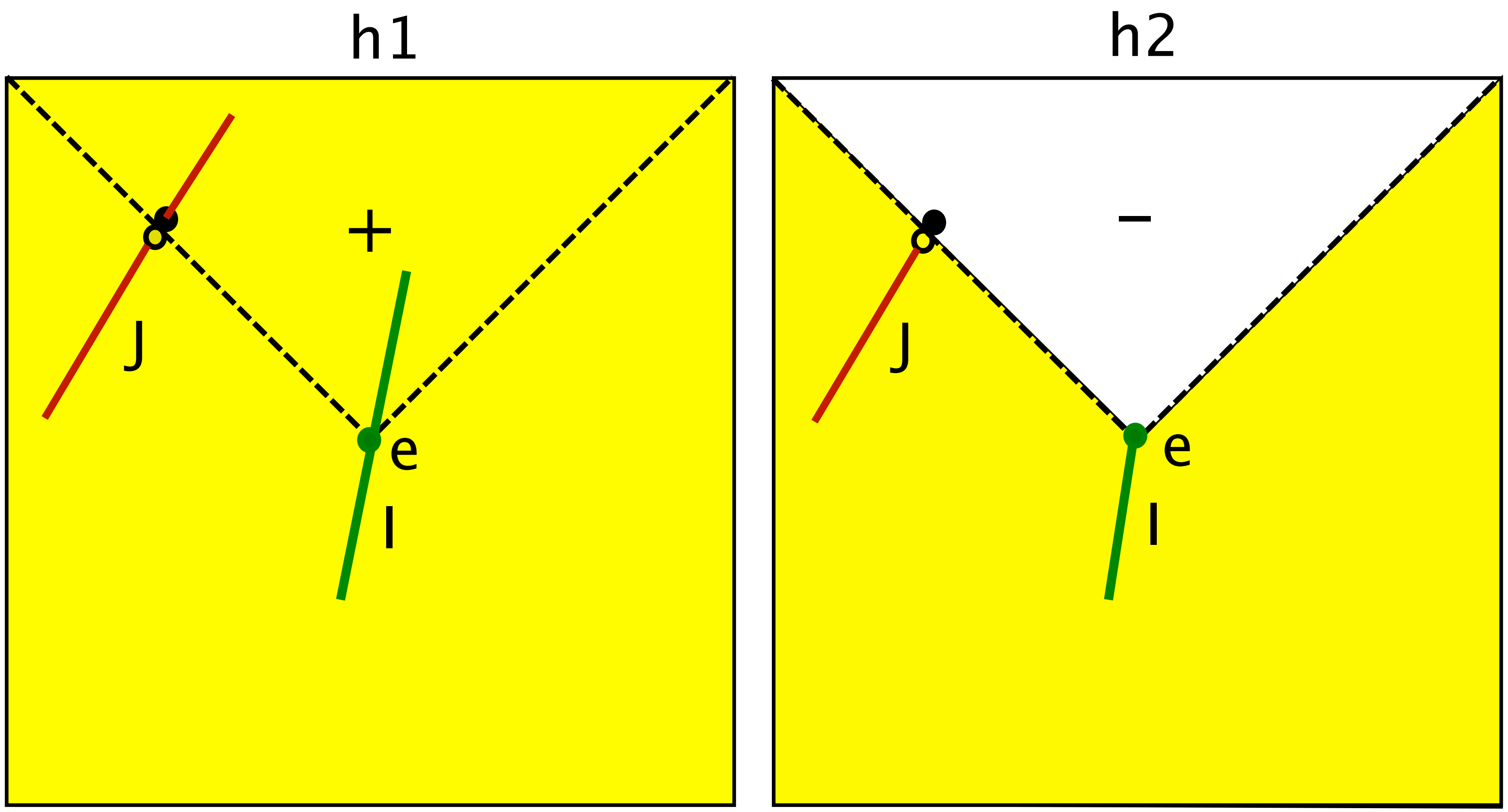


# “Topology of the future light cones”




Chanciness vs. indeterminism without choice

upper bounded chain may have no supremum

Postulates:

upper bounded chain has a supremum in every history it is a subset of

lower bounded chain has an infimum

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$\langle W, \leqslant \rangle$ is a model of BST if $W$ is a nonempty set and $\leqslant$ is a partial ordering on $W$ and the following postulates are satisfied:

1. The ordering $\leqslant$ is dense.
2. $W$ has no maximal elements with respect to $\leqslant$.
3. Every lower bounded chain in $W$ has an infimum in $W$.
4. Every upper bounded chain in $W$ has a supremum in every history that contains it.
5. Prior choice principle (PCP): For any lower bounded chain $O \in h_1 - h_2$ there exists a point $e \in W$ such that $e$ is maximal in $h_1 \cap h_2$ and $\forall e' \in O \; e < e'$.

# Important consequence:

Undividedness of histories at an event is an equivalence relation

$h_1 \equiv_e h_2$ iff (1) $e \in h_1 \cap h_2$ and (2) $\exists e' : (e < e' \wedge e' \in h_1 \cap h_2)$

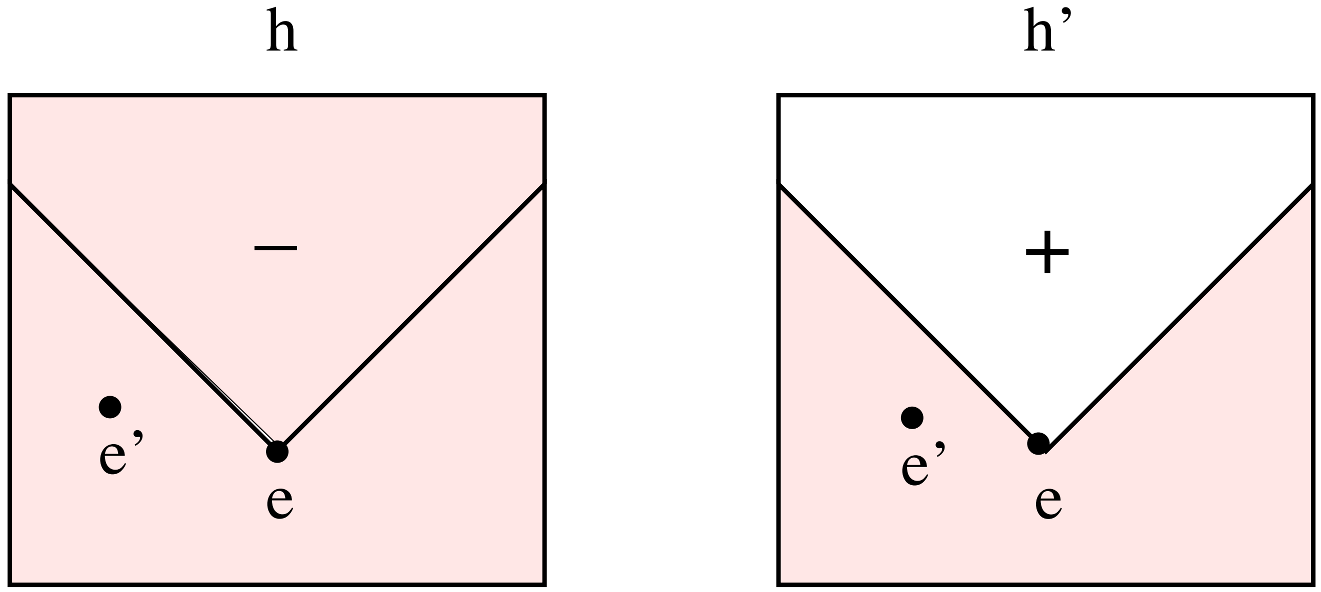
It is an equivalence r. on $H_e := \{h \in Hist \mid e \in h\}$

So it induces partition $\Pi_e$ of $H_e$.

If $h_1, h_2 \in H \in \Pi_e$, then $h_1 \equiv_e h_2$.

Elements of $\Pi_e$ are called "possibilities open at $e$".

# Analysis of non-locality (without probabilities)



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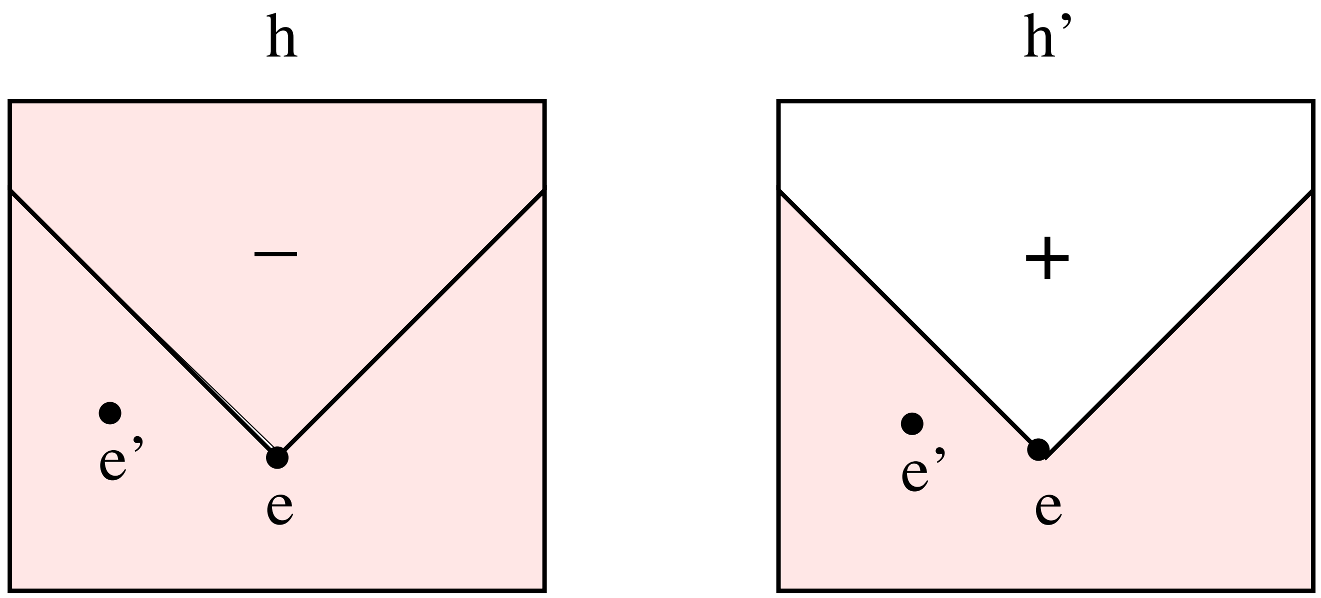


Above: one choice point, below: two choice poins. Smooth combinatorics.

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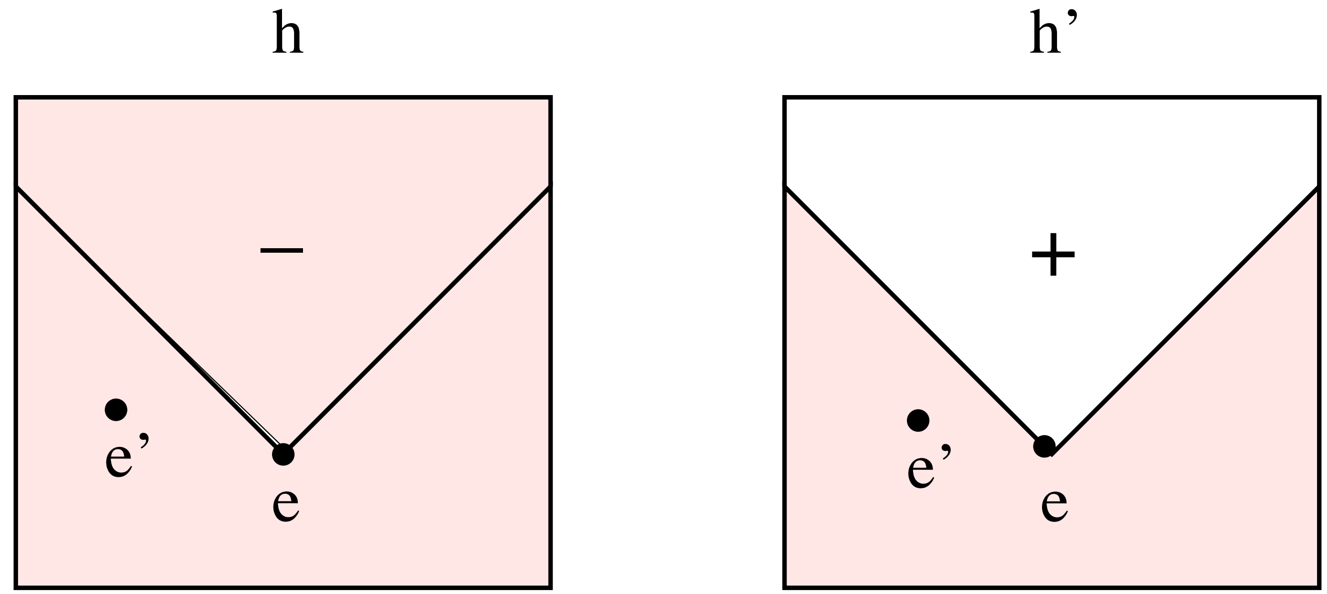


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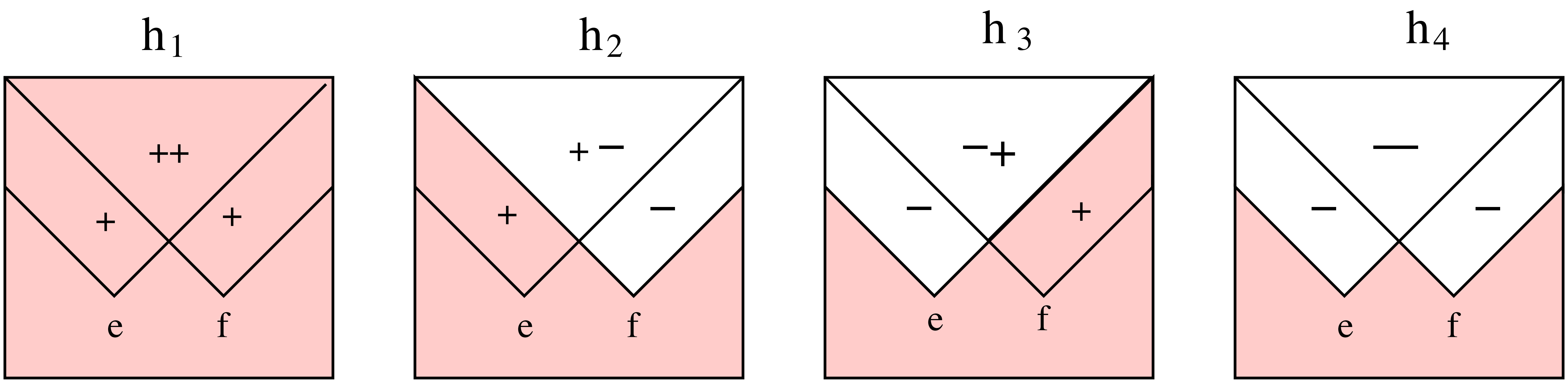
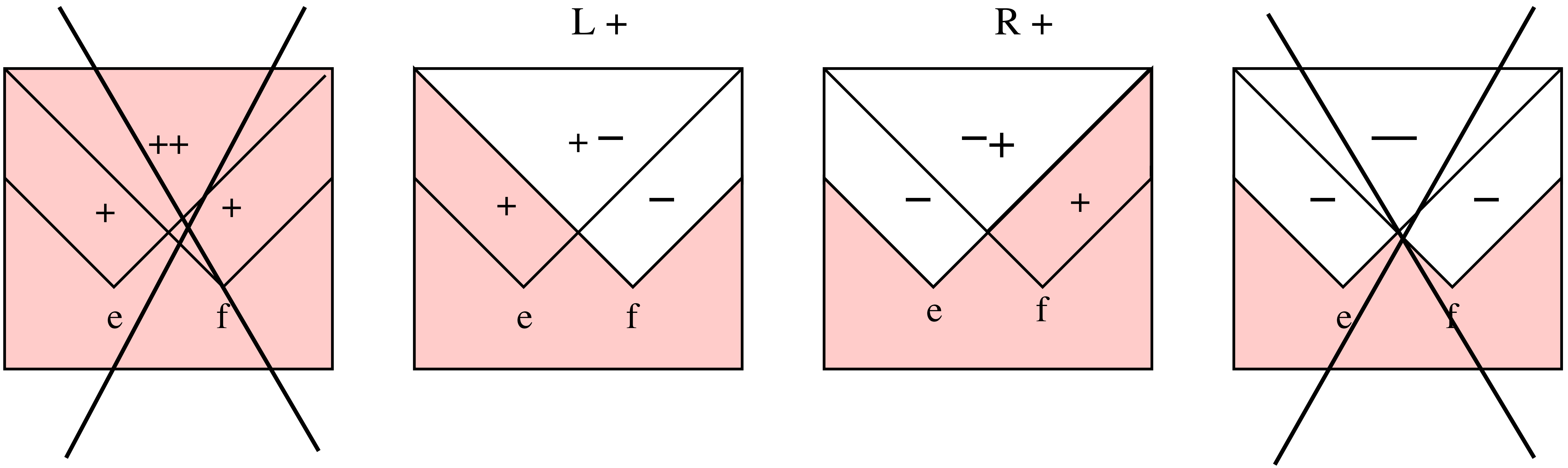


# But smooth combinatorics can fail (EPR)

Combinatorically allowable histories are not possible.




$$\Pi_e = \{\{L+\}, \{R+\}\}$$

$$\Pi_f = \{\{L+\}, \{R+\}\}$$

# Non-locality (or modal funny business)

$e_1$ and $e_2$ are space-like related (SLR) if they are incomparable, yet there is a history to which they both belong.

$H \in \Pi_e$ and $G \in \Pi_f$ constitute a case of modal funny business iff

(1) $e$ SLR $f$, and

(2) $H \cap G = \emptyset$.

# More applications of BST

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1. Analysis of Bell's theorems

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1. Analysis of Bell's theorems
2. Analysis of causation
3. A theory of single case objective probabilities (chances)
4. Analysis of flow of time (see tomorrow)
5. A theory of agency: our actions and their consequences

# Analysis of Bell's theorems *aka* modal and probabilistic funny business

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